In all these experiments, the pipes employed were open at both ends.

Now that science is in possession of this delicate optical method, which requires for its success no nice musical ear, other problems, heretofore settled by assumption, may be brought within the range of demonstration.

Five hundred and eighty-seventh Meeting.

November 13, 1867. — Statute Meeting.

The PRESIDENT in the chair.

The President announced the decease of Professor Mitter-maier, of the Foreign Honorary Members.

Professor Lovering announced that Vol. IX. Part I. of the Memoirs, was ready for distribution.

Professor Edward C. Pickering was elected a Resident Fellow in Class I. Section 3.

Dr. C. H. F. Peters was elected an Associate Fellow in Class I. Section 2.

On the motion of Dr. G. E. Ellis, the Rumford Committee was instructed to collect papers relating to the life of Count Rumford.

The following paper was presented : —

Upon Logical Comprehension and Extension. By C. S. PEIRCE.

§ 1. *That these Conceptions are not so Modern as has been represented.*

THE historical account usually given of comprehension and extension is this, " that the distinction, though taken in general terms by Aristotle, and explicitly announced with scientific precision by one, at least, of his Greek commentators, had escaped the marvellous acuteness of the schoolmen, and remained totally overlooked and forgotten till the publication of the Port Royal Logic." *   I would offer

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* This is quoted from Baines (Port Royal Logic, 2d ed. p. xxxiii.), who says that he is indebted to Sir William Hamilton for the information.

the following considerations to show that this interpretation of history is not exactly true. In the first place, it is said that a distinction was taken between these attributes, as though they were previously confounded. Now there is not the least evidence of this. A German logician, has, indeed, by a subtle misconception, considered extension as a species of comprehension, but, to a mind beginning to reflect, no notions seem more unlike. The mental achievement has been the bringing of them into relation to one another, and the conception of them as factors of the import of a term, and not the separation of them. In the second place it is correctly said that the doctrine taught by the Port Royalists is substantially contained in the work of a Greek commentator. That work is no other than Porphyry's Isagoge * ; and therefore it would be most surprising if the doctrine had been totally overlooked by the schoolmen, for whether their acuteness was as marvellous as Hamilton taught or not, they certainly studied the commentary in question as diligently as they did the Bible. It would seem, indeed, that the tree of Porphyry involves the whole doctrine of extension and comprehension except the names. Nor were the scholastics without names for these quantities. The *partes subjectives* and *partes essentiales* are frequently opposed; and several other synonymes are mentioned by the Conimbricenses. It is admitted that Porphyry fully enunciates the doctrine; it must also be admitted that the passage in question is fully dealt with and correctly explained by the mediæval commentators. The most that can be said, therefore, is that the doctrine of extension and comprehension was not a prominent one in the mediæval logic. †

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* Porphyry appears to refer to the doctrine as an ancient one.

† The author of " De Generibus et Speciebus " opposes the *integral* and *diffinitive* wholes. John of Salisbury refers to the distinction of comprehension and extension, as something " quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa *significant*, et aliud esse quod *nominant*. Nominantur singularia, sed universalia significantur." (Metalogicus, lib. 2, cap. 20. Ed. of 1620, p. 111.)

Vincentius Bellovacensis (*Speculum Doctrinale*, Lib. III. cap. xi.) has the following : " Si vero quæritur utrum hoc universale ' homo ' sit in quolibet homine secundum se totum an secundem partem, dicendum est quod secundum se totum, id est secundum quamlibet sui partem diffinitivam. . . . . Non autem secundum quamlibet partem subjectivam." William of Auvergne (Prantl's *Geschichte*, Vol. III. p. 77) speaks of "totalitatem istam, quæ est ex partibus rationis seu diffinitionis, et hae partes sunt genus et differentiæ ; alio modo partes speciei individua sunt, quoniam ipsam speciem,cum de eis prædicatur,sibi invicem quodammodo partiunter."

A like degree of historical error is commonly committed in reference to another point which will come to be treated of in this paper, allied, at least, as it is most intimately, with the subject of comprehension and extension, inasmuch as it also is founded on a conception of a term as a whole composed of parts, — I mean the distinction of clear and distinct. Hamilton tells us "we owe the discrimination to the acuteness of the great Leibniz. By the Cartesians the distinction had not been taken ; though the authors of the Port Royal Logic came so near that we may well marvel how they failed explicitly to enounce it." (Lectures on Logic ; Lecture IX.)   Now, in fact, all that the Port Royalists say about this matter * is copied from Descartes,† and their variations from his wording serve only to confuse what in him is tolerably distinct.   As for Leibniz, he himself expressly avows that the distinction drawn by Descartes is the same as his own. ‡   Nevertheless, it is very much more clear with Leibniz than with Descartes. A philosophical distinction emerges gradually into consciousness ; there is no moment in history before which it is altogether unrecognized, and after which it is perfectly luminous.   Before Descartes, the distinction of confused and distinct had been thoroughly developed, but the difference between distinctness and clearness is uniformly overlooked. Scotus distinguishes between conceiving confusedly and conceiving the confused, and since any obscure concept necessarily includes more than its proper object, there is always in what is obscurely conceived a conception of something confused ; but the schoolmen came no nearer than this to the distinction of Descartes and Leibniz.

§ 2. *Of the Different Terms applied to the Quantities of Extension and Comprehension.*

Extension and comprehension are the terms employed by the Port Royalists.   Owing to the influence of Hamilton, *intension* is now frequently used for comprehension ; but it is liable to be confounded with intensity, and therefore is an objectionable word.   It is derived from the use of cognate words by Cajetan and other early writers.   *External* and *internal quantity* are the terms used by many early Kantians.

If we were to go to later authors, the examples would be endless.   See any commentary in Phys. Lib. I.

   * Part I. chap. ix.   † *Principia*, Part I. § 45 *et seq.*

   ‡ Eighth Letter to Burnet.

Scope and *force* are proposed by De Morgan.   *Scope* in ordinary language expresses extension, but *force* does not so much express comprehension as the power of creating a lively representation in the mind of the person to whom a word or speech is addressed.   Mr. J. S. Mill has introduced the useful verbs *denote* and *connote*, which have become very familiar.   It has been, indeed, the opinion of the best students of the logic of the fourteenth, fifteenth, and sixteenth centuries, that *connotation* was in those ages used exclusively for the reference to a second significate, that is (nearly) for the reference of a relative term (such as *father*, *brighter*, &c.) to the correlate of the object which it primarily denotes, and was never taken in Mill's sense of the reference of a term to the essential characters implied in its definition. §   Mr. Mill has, however, considered himself entitled to deny this upon his simple authority, without the citation of a single passage from any writer of that time.   After explaining the sense in which he takes the term *connote*, he says : " The schoolmen, to whom we are indebted for the greater part of our logical language, gave us this also, and in this very sense. For though some of their general expressions countenance the use of the word in the more extensive and vague acceptation in which it is taken by Mr. [James] Mill, yet when they had to define it specifically as a technical term, and to fix its meaning as such, with that admirable precision which always characterized their definitions, they clearly explained that nothing was said to be connoted except *forms*, which word may generally, in their writings, be understood as synonymous with *attributes*." As scholasticism is usually said to come to an end with Occam, this conveys the idea that *connote* was commonly employed by earlier writers.   But the celebrated Prantl considers it conclusive proof that a passage in Occam's *Summa* is spurious, that *connotative* is there spoken of as a term in frequent use ; * and remarks upon a passage of Scotus in which *connotatum* is found, that this conception is here met with for the first time. †   The term occurs, however, in Alexander of Ales, ‡ who makes *nomen connotans* the equivalent of *appellatio relativa*, and takes the relation itself as the object of *connotare*, speaking of creator as connoting the relation of

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* Prantl, *Geschichte*, Vol. III. p. 364.

† Ibid. p. 134.   Scotus also uses the term. *Quodlib*. question 13, article 4.

‡ *Summa Theologica*, Part I. question 53.

§ Cf. Morin, *Dictionnaire*, Tome I. col. 684 ; Chauvin, *Lexicon*, both editions ; Eustachius, *Summa*, Part I. Tr. I. qu. 6.

creator to creature. Occam's *Summa* * contains a chapter devoted to the distinction of absolute and connotative names. The whole deserves to be read, but I have only space to quote the following: " Nomen autem connotativum est illud quod significat aliquid primario et aliquid secundario; et tale nomen proprie habet diffinitionem exprimentem quid nominis et frequenter oportet ponere aliquid illius diffinitionis in recto et aliud in obliquo; sicut est de hoc nomine album, nam habet diffinitionem exprimentem quid nominis in qua una dictio ponitur in recto et alia in obliquo. Unde si queratur quid significat hoc nomen album, dices quod idem quod illa oratio tota ' aliquid informatum albedine' vel 'aliquid habens albedinem' et patet quod una pars orationis istius ponitur in recto et alia in obliquo. . . . . Huiusmodi autem nomina connotativa sunt omnia nomina concreta primo modo dicta, et hoc *quia talia concreta significant unum in recto et aliud in obliquo,* hoc est dictu, in diffinitione exprimente quid nominis debet poni unus rectus significans unam rem et alius obliquus significans aliam rem, sicut patet de omnibus talibus, iustus, albus, animatus, et sic de aliis. Huiusmodi etiam nomina sunt omnia nomina relatiua, quia semper in eorum diffinitionibus ponuntur diversa idem diuersis modis vel diuérsa significantia, sicut patet de hoc nomine simile. Mere autem absoluta sunt illa quæ non significant aliquid principaliter et aliud vel idem secundario, sed quicquid significatur per tale nomen æque primo significatur sicut patet de hoc nomine animal." Eckius, in his comment on Petrus Hispanus, has also some extended remarks on the signification of the term *connote,* which agree in the main with those just quoted.† Mr. Mill's historical statement cannot, therefore, be admitted.

Sir William Hamilton has borrowed from certain late Greek writers the terms *breadth* and *depth,* for extension and comprehension respectively. ‡ These terms have great merits. They are brief; they are suited to go together; and they are very familiar. Thus, " wide" learning is, in ordinary parlance, learning of many things; " deep" learning, much knowledge of some things. I shall, therefore, give the preference to these terms. Extension is also called *sphere* and *circuit ;* and comprehension, *matter* and *content.*

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* Part I. chap. X. (Ed. of 1488, fol. 6, c.)

† Fol. 23. d. See also Tatareti Expositio in Petr. Hisp. towards the end. Ed. of 1509, fol. 91, b.

‡ Logic, p. 100. In the *Summa Logices* attributed to Aquinas, we read: " Omnis forma sub se habens multa, idest quod universaliter sumitur, habet quandam *latitudinem ;* nam invenitur in pluribus, et dicitur de pluribus." (Tr. 1, c. 3.)

§ 3. *Of the Different Senses in which the Terms Extension and Comprehension have been accepted.*

The terms *extension* and *comprehension,* and their synonymes, are taken in different senses by different writers. This is partly owing to the fact that while most writers speak only of the extension and comprehension of concepts, others apply these terms equally to concepts and judgments (Rösling), others to any mental representation (Überweg and many French writers), others to cognition generally (Baumgarten), others to "terms" (Fowler, Spalding), others to names (Shedden), others to words (McGregor), others to "meanings" (Jevons), while one writer speaks only of the extension of *classes* and the comprehension of *attributes* (De Morgan in his Syllabus).

Comprehension is defined by the Port Royalists as "those attributes which an idea involves in itself, and which cannot be taken away from it without destroying it."

It will be remembered that the *marks* of a term are divided by logicians first into the necessary and the accidental, and that then the necessary marks are subdivided into such as are strictly essential, that is, contained in the definition, and such as are called proper. Thus it is an essential mark of a triangle to have three sides; it is a proper mark to have its three angles equal to two right angles; and it is an accidental mark to be treated of by Euclid.

The definition of the Port Royalists, therefore, makes comprehension include all necessary marks, whether essential or proper.

The Port Royalists attribute comprehension immediately to any ideas. Very many logicians attribute it immediately only to concepts. Now a concept, as defined by them, is strictly only the essence of an idea ; they ought therefore to include in the comprehension only the essential marks of a term. These logicians, however, abstract so entirely from the real world, that it is difficult to see why these essential marks are not at the same time all the marks of the object as they suppose it.

There can, I think, be no doubt that such writers as Gerlach and Sigwart make comprehension include all marks, necessary or accidental, which are universally predicable of the object of the concept.

Again, most German writers regard the comprehension as a sum either of concepts (Drobisch, Bachmann, etc.) or of elements of intuition (Trendelenburg). But many English writers regard it as the

sum of real external attributes (Shedden, Spalding, Devey, De Morgan, Jevons, McGregor, Fowler).

According to most writers, comprehension consists of the (necessary) attributes *thought* as common to the objects.   Shedden defines it as consisting of all the attributes common to the things denoted.

Again, most logicians consider as marks only such as are virtually * predicated ; a few, perhaps, only such as are actually thought, and still fewer include those which are habitually thought.   Here and there is found an author who makes comprehension include all true attributes, whether thought or not.

There is also a difference in the mode of reckoning up the marks. Most writers count all distinguishable marks, while a few consider co-extensive marks as the same.

In the use of the term " extension " the want of a definite convention is still more marked.   The Port Royalists define it as " those *subjects* to which the idea applies."   It would appear, therefore, that it might include mere fictions.

Others limit the term to *real* species, and at the same time extend it to single beings.   This is the case with Watts, and also with Friedrich Fischer.

Others are most emphatic in declaring that they mean by it *things*, and not species, real or imaginary.   This is the case with Bachmann, Esser, and Schulze.

Others make it include neither concepts nor things, but singular representations.   This is the case with the strict Kantian.

The following table exhibits this diversity : —

Extension embraces

Individual representations	according to	Kant, E. Reinhold, etc.
Representations	" "	Fries, Überweg, etc.
Real external things and species	" "	Watts, Shedden, etc.
Real external individual objects	" "	Bachmann, Devey, etc.
Things	" "	Schulze, Bowen, etc.
Species	" "	Drobisch, De Morgan, etc.
Objects (representations)	" "	Thomson, etc.
Individuals	" "	Mahan.
Concepts	" "	Herbart, Vorländer, etc.
General terms	" "	Spalding.
Psychical concepts	" "	Strümpell.
Variable marks	" "	Ritter.

* I adopt the admirable distinction of Scotus between actual, habitual, and virtual cognition.

Again, logicians differ as to whether by extension they mean the concepts, species, things, or representations to which the term is habitually applied in the judgment, or all to which it is truly applicable. The latter position is held by Herbart, Kiesewetter, etc. ; the former by Duncan, Spalding, Vorländer, Überweg, etc.

Some logicians include only *actual* things, representations, etc., under extension (Bachmann, Fries, Herbart) ; others extend it to such as are merely possible (Esser, Ritter, Gerlach).

Finally, some few logicians speak of the two quantities as numerical, while most writers regard them as mere aggregates of diverse objects or marks.

§ 4. *Denials of the Inverse Proportionality of the two Quantities, and Suggestions of a third Quantity.*

Until lately the law of the inverse proportionality of extension and comprehension was universally admitted. It is now questioned on various grounds.

Drobisch says that the comprehension varies arithmetically, while the extension varies geometrically. This is true, in one sense.

Lotze, after remarking that the only conception of a universal which we can have is the power of imagining singulars under it, urges that the possibility of determining a concept in a way corresponding to each particular under it is a mark of that concept, and that therefore the narrower concepts have as many marks as the wider ones. But, I reply, *these* marks belong to the concept in its second intention, and are not common marks of those things to which it applies, and are therefore no part of the comprehension. They are, in fact, the very marks which constitute the extension. No one ever denied that extension is a mark of a concept; only it is a certain mark of second intention.

Vorländer's objection is much more to the purpose. It is that if from any determinate notion, as that of Napoleon, we abstract all marks, all determination, what remains is merely the conception *something*, which has no more extension than Napoleon. " Something" has an uncertain sphere, meaning either this thing or that or the other, but has no general extension, since it means one thing only. Thus, before a race, we can say that some horse will win, meaning this one, that one, or that one; but by some horse we mean but one, and it therefore has no more extension than would a term definitely indicating which, — although this latter would be more determinate, that is, would have

more comprehension. I am not aware that those who adhere to Kant's unmodified doctrine have succeeded in answering this objection.

Überweg has the following remarks.* " To the higher representation, since conformably to its definition it contains only the common elements of content of several lower representations, belongs in comparison to each of the lower a more limited content, but a wider circuit. The lower representation, on the contrary, has a richer content but narrower circuit. Yet by no means by every diminution or increase of a given content does the circuit increase or diminish, nor by every increase or diminution of a given circuit does the content diminish or increase." I am surprised that he does not explain himself further upon this point, which it is the principal object of this paper to develop.

De Morgan says : † " According to such statements as I have seen, ' man residing in Europe, drawing breath north of the equator, seeing the sun rise before those in America,' would be a more intensively quantified notion than ' man residing in Europe ' ; but certainly not less *extensive*, for the third and fourth elements of the notion must belong to those men to whom the first and second belong." Mr. De Morgan adopts the definitions of extension and comprehension given by the Port Royalists. According to those definitions, if the third and fourth elements necessarily belong to the notion to which the first and second belong, they are parts of the comprehension of that second notion which is composed of the first and second elements, and therefore the two notions are equal in comprehension ; but if this is not the case, then the second notion can be predicated of subjects of which the first cannot, for example, of " man residing in Europe drawing breath south of the Equator " ; for that there is really no such man will not affect the truth of the proposition, and therefore the second notion is more extensive than the first.

Two logicians, only, as far as I remember, Archbishop Thomson ‡ and Dr. W. D. Wilson, § while apparently admitting Kant's law, wish to establish a third quantity of concepts. Neither gentleman has defined his third quantity, nor has stated what its relations to the other two are. Thomson calls his Denomination. It seems to be the same as Extension regarded in a particular way. Dr. Wilson terms his new quantity Protension ; it has something to do with time, and appears to be generally independent of the other two. It is plain, indeed, that as

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* Logik, 2te Aufl. § 54.   ‡ Laws of Thought, 4th ed., §§ 52, 80.
† Formal Logic, p. 234. His doctrine is different in the Syllabus.
§ Logic, Part I. chap. ii. § 5.

long as Kant's law holds, and as long as logical quantities can only be compared as being more or less and not directly measured, and as long as the different *kinds* of quantity cannot be compared at all, a third quantity must be directly proportional to one or other of the known quantities, and therefore must measure the same thing, or else must be independent of the other two, and be quite unconnected with them.

§ 4. *Three Principal Senses in which Comprehension and Extension will be taken in this Paper.*

I shall adopt Hamilton's terms, *breadth* and *depth*, for extension and comprehension respectively, and shall employ them in different senses, which I shall distinguish by different adjectives.

By the *informed breadth* of a term, I shall mean all the real things of which it is predicable, with logical truth on the whole in a supposed state of information. By the phrase " on the whole" I mean to indicate that all the information at hand must be taken into account, and that those things of which there is not on the whole reason to believe that a term is truly predicable are not to be reckoned as part of its breadth.

If T be a term which is predicable only of S', S'', and S''', then the S' 's, the S'' 's, and the S''' 's, will constitute the informed breadth of T. If at the same time, S' and S'' are the subjects of which alone another term T' can be predicated, and if it is not known that all S''' 's are either S' or S'', then T is said to have a greater informed breadth than T'. If the S''' 's are known not to be all among the S' 's and S'' 's, this excess of breadth may be termed *certain*, and, if this is not known, it may be termed *doubtful*. If there are known to be S''' 's, not known to be S' 's or S'' 's, T is said to have a greater *actual* breadth than T' ; but if no S''' 's are known except such are known to be S' 's, and S'' 's (though there may be others), T is to have a greater *potential* breadth than T'. If T and T' are conceptions in different minds, or in different states of the same mind, and it is known to the mind which conceives T that every S''' is either S'' or S', then T is said to be more *extensively distinct* than T'.*

By the informed depth of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it † (with

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* For the distinction of extensive and comprehensive distinctness, see Scotus, i. dist. 2 . qu. 3.

† That is, of whatever things it is applicable to.

logical truth, on the whole) in a supposed state of information ; no character being counted twice over knowingly in the supposed state of information.   The depth, like the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness.

The informed breadth and depth suppose a state of information which lies somewhere between two imaginary extremes. These are, first, the state in which no fact would be known, but only the meaning of terms ; and, second, the state in which the information would amount to an absolute intuition of all there is, so that the things we should know would be the very substances themselves, and the qualities we should know would be the very concrete forms themselves.   This suggests two other sorts of breadth and depth corresponding to these two states of information, and which I shall term respectively the *essential* and the *substantial* breadth and depth.

By the *essential depth* of a term, then, I mean the really conceivable qualities predicated of it in its definition.

The defined term will not perhaps be applicable to any real objects whatever.   Let, for example, the definition of the term T be this,

$$\text{Any T is both P' and P'' and P''',}$$

then this sums up its whole meaning; and, as it may not be known that there is any such thing as P', the meaning of T does not imply that it exists.   On the other hand, we know that neither P', P'', nor P''' is coextensive with the whole sphere of being.   For they are determinate qualities, and it is the very meaning of being that it is indeterminate, that is, is more extensive than any determinate term. In fact, P', for example, is a real notion which we never could have except by means of its contrast to something else.   Hence we must know that

$$\text{Whatever is not-P' is not-T,}$$

$$\text{Whatever is not-P'' is not-T,}$$

and   Whatever is not-P''' is not T.

Thus if we define the *essential breadth* of a term as those real things of which, according to its very meaning, a term is predicable, not-T has an essential breadth.   We may therefore divide all terms into two classes, the essentially affirmative or positive and the essentially

negative ; of which the former have essential depth, but no essential breadth, and the latter essential breadth, but no essential depth. It must be noted, however, that this division is not the same as the similar one which language makes. For example, *being*, according to this, is an essentially negative term, inasmuch as it means that which can be predicated of whatever you please, and so has an essential breadth ; while *nothing* is an essentially positive term, inasmuch as it means that of which you are at liberty to predicate what you please, and therefore has an essential depth. The essential subjects of being cannot be enumerated, nor the essential predicates of nothing.

In essential breadth or depth, no two terms can be equal; for, were that the case, the two terms would have the same meaning, and therefore, for logical purposes, would be the same term. Two terms may have unknown relations in these quantities, on account of one or other of them not being distinctly conceived.

Substantial breadth is the aggregate of real substances of which alone a term is predicable with absolute truth. *Substantial depth* is the real concrete form which belongs to everything of which a term is predicable with absolute truth.

General terms denote several things. Each of these things has in itself no qualities, but only a certain concrete form which belongs to itself alone. This was one of the points brought out in the controversy in reference to the nature of universals.* As Sir William Hamilton says, not even the humanity of Leibniz belongs to Newton, but a different humanity. It is only by abstraction, by an oversight, that two things can be said to have common characters. Hence, a general term has no *substantial depth*. On the other hand, particular terms, while they have *substantial depth*, inasmuch as each of the things, one or other of which are predicated of them, has a concrete form, yet have no *substantial breadth*, inasmuch as there is no aggregate of things to which alone they are applicable. In order to place this matter in a clearer light, I must remark, that I, in common with most logicians, take the copula in the sense of a sign of attribution, and not, like Hamilton, in the sense of a sign of equality in extension or comprehension. He exposes the proposition, " man is an animal," thus : —

The extension of man . . . . *Subject.*
equals . . . . *Copula.*
a part or all of the extension of animal . . . . *Predicate.*

* See, for example, *De Generibus et Speciebus*, p. 548.

And thus he makes the predicate particular. Others interpret it thus : —

Every man	*Subject.*
has all the attributes common to	*Copula.*
every animal	*Predicate.*

It is in this latter sense that the copula is considered in this paper. Now, a particular is, as has been said, an *alternative* subject. Thus, "Some S is M" means, if S', S'', and S''' are the singular S's, that "either S', or else S'', or else S''', has all the attributes belonging to M." A particular term, then, has a substantial depth, because it may have a predicate which is absolutely concrete, as in the proposition, " Some man is Napoleon." But if we put the particular into the predicate we have such a proposition as this : " M has all the attributes belonging to S', or else all those belonging to S'', or else all those belonging to S'''." And this can never be true unless M is a single individual. Now a single individual substance is, I will not say an atom, but the smallest part of an atom, that is, nothing at all. So that a particular can have no *substantial breadth*. Now take the universal term " S." We can say, " Any S is M," but not if M is a real concrete quality. We cannot say, for instance, " Any man is Napoleon." On the other hand, we can say " Any M is S," even if M is a real substance or aggregate of substances. Hence a universal term has no *substantial depth*, but has *sub-. stantial breadth*. We may therefore divide all terms into substantial universals and substantial particulars.

Two terms may be equal in their substantial breadth and depth, and differ in their essential breadth and depth. But two terms cannot have relations of substantial breadth and depth which are unknown in the state of information supposed, because in that state of information everything is known.

In informed breadth and depth, two terms may be equal, and may have unknown relations. Any term, affirmative or negative, universal or particular, may have informed breadth or depth.

§ 5. *The Conceptions of Quality, Relation, and Representation, applied to this Subject.*

In a paper presented to the Academy last May, I endeavored to show

that the three conceptions of reference to a ground, reference to a correlate, and references to an interpretant, are those of which logic must principally make use. I there also introduced the term "symbol," to include both concept and word. Logic treats of the reference of symbols in general to their objects. A symbol, in its reference to its object, has a triple reference: —

1st., Its direct reference to its object, or the real things which it represents;

2d., Its reference to its ground through its object, or the common characters of those objects;

3d., Its reference to its interpretant through its object, or all the facts known about its object.

What are thus referred to, so far as they are known, are: —

1st., The informed *breadth* of the symbol;

2d., The informed *depth* of the symbol;

3d., The sum of synthetical propositions in which the symbol is subject or predicate, or the *information* concerning the symbol.

By breadth and depth, without an adjective, I shall hereafter mean the informed breadth and depth.

It is plain that the breadth and depth of a symbol, so far as they are *not* essential, measure the *information* concerning it, that is, the synthetical propositions of which it is subject or predicate. This follows directly from the definitions of breadth, depth, and information. Hence it follows: —

1st., That, as long as the information remains constant, the greater the breadth, the less the depth;

2d., That every increase of information is accompanied by an increase in depth or breadth, independent of the other quantity;

3d., That, when there is no information, there is either no depth or no breadth, and conversely.

These are the true and obvious relations of breadth and depth. They will be naturally suggested if we term the information the *area*, and write —

$$\text{Breadth} \times \text{Depth} = \text{Area}.$$

If we learn that S is P, then, as a general rule, the depth of S is increased without any decrease of breadth, and the breadth of P is increased without any decrease of depth. Either increase may be *certain* or *doubtful*.

It may be the case that either or both of these increases does not take place. If P is a negative term, it may have no depth, and therefore adds nothing to the depth of S. If S is a particular term, it may have no breadth, and then adds nothing to the breadth of P. This latter case often occurs in metaphysics, and, on account of not-P as well as P being predicated of S, gives rise to an appearance of contradiction where there really is none ; for, as a contradiction consists in giving to contradictory terms some breadth in common, it follows that, if the common subject of which they are predicated has no real breadth, there is only a verbal, and not a real contradiction. It is not really contradictory, for example, to say that a boundary is both within and without what it bounds. There is also another important case in which we may learn that " S is P," without thereby adding to the depth of S or the breadth of P. This is when, in the very same act by which we learn that S is P, we also learn that P was covertly contained in the previous depth of S, and that consequently S was a part of the previous breadth of P. In this case, P gains in extensive distinctness and S in comprehensive distinctness.

We are now in condition to examine Vorländer's objection to the inverse proportionality of extension and comprehension. He requires us to think away from an object all its qualities, but not, of course, by thinking it to be without those qualities, that is, by denying those qualities of it in thought. How then ? Only by supposing ourselves to be ignorant whether it has qualities or not, that is, by diminishing the supposed information ; in which case, as we have seen, the depth can be diminished without increasing the breadth. In the same manner we can suppose ourselves to be ignorant whether any American but one exists, and so diminish the breadth without increasing the depth.

It is only by confusing a movement which is accompanied with a change of information with one which is not so, that people can confound generalization, induction, and abstraction. *Generalization* is an increase of breadth and a decrease of depth, without change of information. *Induction* is a certain increase of breadth without a change of depth, by an increase of believed information. *Abstraction* is a decrease of depth without any change of breadth, by a decrease of conceived information. *Specification* is commonly used (I should say unfortunately) for an increase of depth without any change of breadth, by an increase of asserted information. *Supposition* is used for the

same process when there is only a conceived increase of information. *Determination*, for any increase of depth. *Restriction*, for any decrease of breadth; but more particularly without change of depth, by a supposed decrease of information. *Descent*, for a decrease of breadth and increase of depth, without change of information.

Let us next consider the effect of the different kinds of reasoning upon the breadth, depth, and area of the two terms of the conclusion.

In the case of deductive reasoning it would be easy to show, were it necessary, that there is only an increase of the extensive distinctness of the major, and of the comprehensive distinctness of the minor, without any change in information. Of course, when the conclusion is negative or particular, even this may not be effected.

Induction requires more attention. Let us take the following example : —

S', S'', S''', and S^{IV} have been taken at random from among the M's;

S' S'', S''', and S^{IV} are P :

∴ any M is P.

We have here, usually, an increase of information. M receives an increase of depth, P of breadth. There is, however, a difference between these two increases. A new predicate is actually added to M ; one which may, it is true, have been covertly predicated of it before, but which is now actually brought to light. On the other hand, P is not *yet* found to apply to anything but S', S'', S''', and S^{IV}, but only to apply to whatever else may hereafter be found to be contained under M. The induction itself does not make known any such thing. Now take the following example of hypothesis : —

M is, for instance, P', P'', P''', and P^{IV} ;

S is P', P'', P''', and P^{IV} :

∴ S is all that M is.

Here again there is an increase of information, if we suppose the premises to represent the state of information before the inferences. S receives an addition to its depth ; but only a potential one, since there is nothing to show that the M's have any common characters besides P', P'', P''', and P^{IV}. M, on the other hand, receives an actual increase of breadth in S, although, perhaps, only a *doubtful* one. There is, therefore, this important difference between induction and hypothesis, that the former potentially increases the breadth of one term, and actually increases the depth of another, while the latter potentially in-

creases the depth of one term, and actually increases the breadth of another.

Let us now consider reasoning from definition to definitum, and also the argument from enumeration. A defining proposition has a meaning. It is not, therefore, a merely identical proposition, but there is a difference between the definition and the definitum. According to the received doctrine, this difference consists wholly in the fact that the definition is distinct, while the definitum is confused. But I think that there is another difference. The definitum implies the character of being designated by a word, while the definition, previously to the formation of the word, does not. Thus, the definitum exceeds the definition in depth, although only *verbally*. In the same way, any unanalyzed notion carries with it a feeling, — a constitutional word, — which its analysis does not. If this be so, the definition is the predicate and the definitum the subject, of the defining proposition, and this last cannot be simply converted. In fact, the defining proposition affirms that whatever a certain name is applied to is supposed to have such and such characters; but it does not strictly follow from this, that whatever has such and such characters is actually called by that name, although it certainly *might* be so called. Hence, in reasoning from definition to definitum, there is a verbal increase of depth, and an actual increase of extensive distinctness (which is analogous to breadth). The increase of depth being merely verbal, there is no possibility of error in this procedure. Nevertheless, it seems to me proper, rather to consider this argument as a special modification of hypothesis than as a deduction, such as is reasoning from definitum to definition. A similar line of thought would show that, in the argument from enumeration, there is a verbal increase of breadth, and an actual increase of depth, or rather of comprehensive distinctness, and that therefore it is proper to consider this (as most logicians have done) as a kind of infallible induction. These species of hypothesis and induction are, in fact, merely hypotheses and inductions from the essential parts to the essential whole; this sort of reasoning from parts to whole being demonstrative. On the other hand, reasoning from the substantial parts to the substantial whole is not even a probable argument. No ultimate part of matter fills space, but it does not follow that no matter fills space.